Timing Is Everything

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Oxford University

BCS Meeting, Oxford

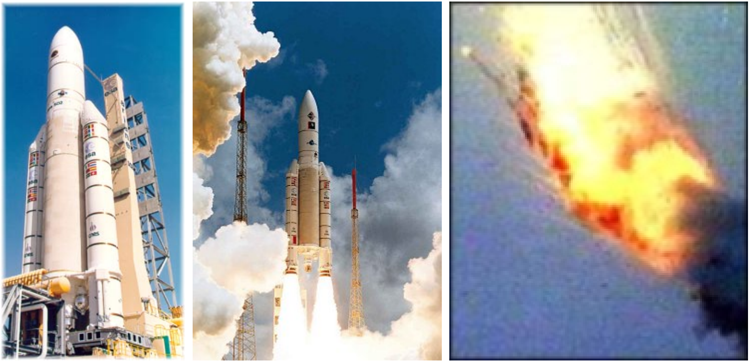
17 May 2012

Automated Verification

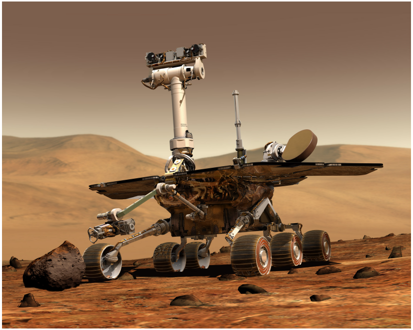
“In theory, there is no difference between theory and practice. In practice, there is.”

Jan L.A. van de Snepscheut

Ariane 5 Explosion, French Guyana, 1996

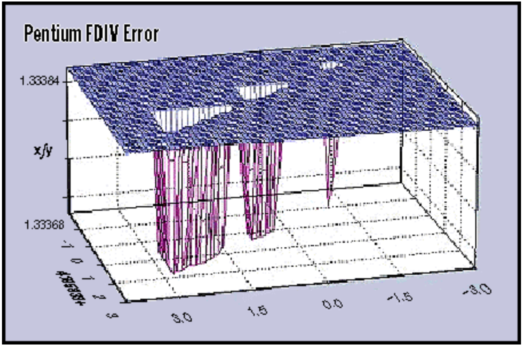


NASA Mars Missions, 1997–2004

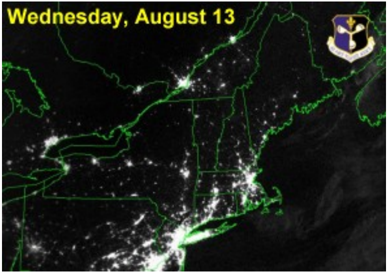


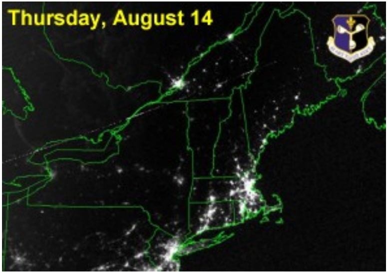
- 1997: Mars Rover loses contact
- 1999: Mars Climate Orbiter is lost
- 1999: Mars Polar Lander is lost
- 2004: Mars Rover freezes

Intel Pentium FDIV Bug, 1994



Northeast Blackout, 2003





Chrysler Pacifica SUV, 2006

December 2006: DaimlerChrysler recalls 128,000 Pacifica sports utility vehicles because of a problem with the software governing the fuel pump and power train control. The defect could cause the engine to stall unexpectedly. [Washington Post]

Automated Verification

"A Grand Challenge for computing research."

Sir Tony Hoare, 2003

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Now one of a small handful of areas *'targetted for growth'* by UK funding council EPSRC.

Automated Verification

“Nobody is going to run into a friend’s office with a program verification. Nobody is going to sketch a verification out on a paper napkin. . . One can feel one’s eyes glaze over at the very thought.”

Rich de Millo, Richard Lipton, Alan Perlis, 1979

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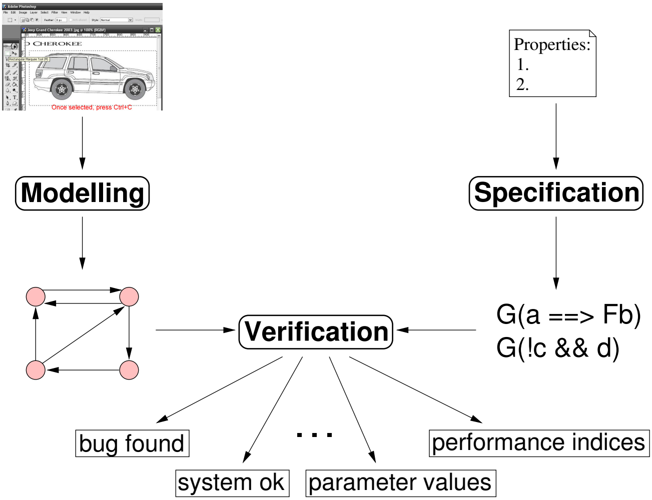
"The success of program verification as a generally applicable and completely reliable method for guaranteeing program performance is not even a theoretical possibility."

James H. Fetzer
Program Verification: The Very Idea, CACM 31(9), 1988

Automated Verification: A High-Level Overview



SLAM

TERMINATOR

proof tools for termination and liveness

TERMINATOR vs. The Ackermann Function

```
int Ack(int m, int n) {
    if (m == 0)
        return n + 1;
    else if (n == 0)
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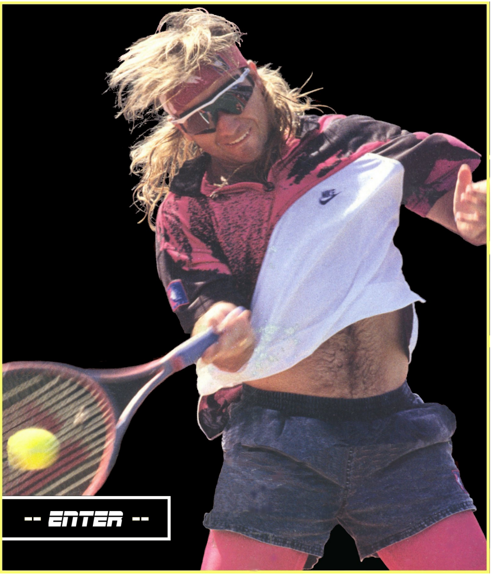
$$\mathrm{Ack}(n,n):\quad 1,\quad 3,\quad 7,\quad 61,\quad 2^{2^{2^{2^{2^{2^{2}}}}}}-3,\quad \underbrace{2^{2^{\cdot^{\cdot^{\cdot^{2}}}}}}_{\mathrm{Ack}(5,4)+3}-3$$

Timing Is Everything



A Login Protocol

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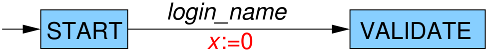

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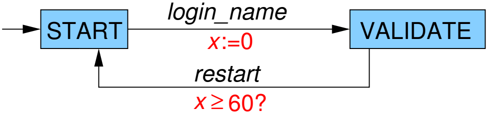

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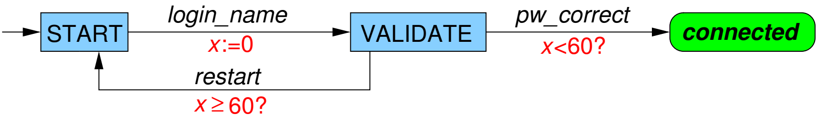

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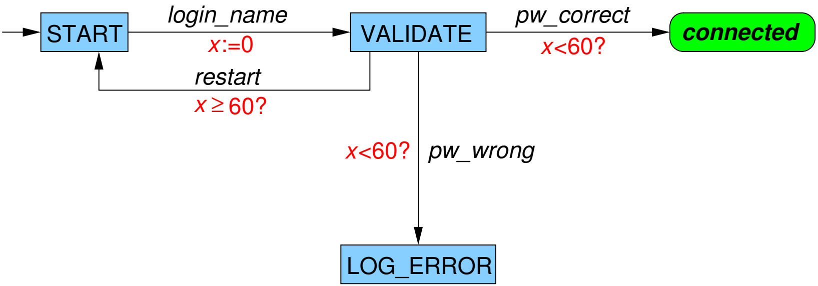

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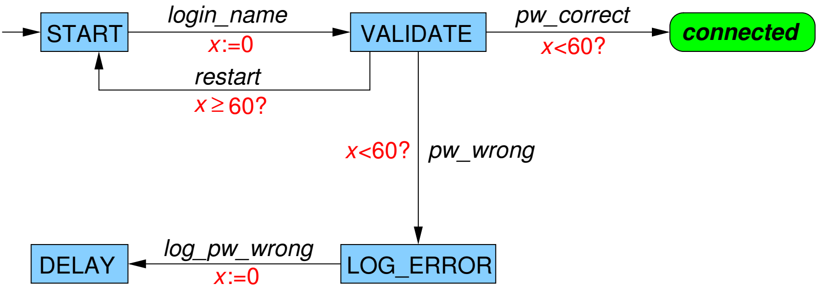

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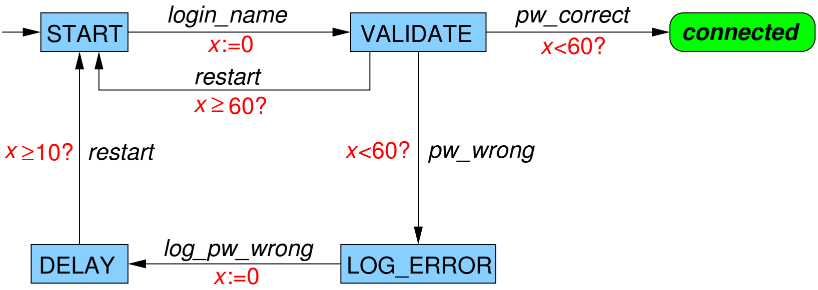
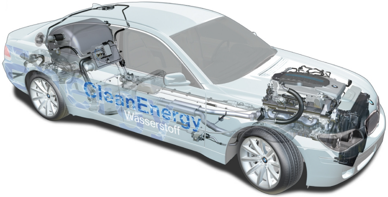


BMW Hydrogen 7



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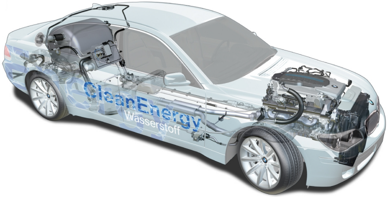

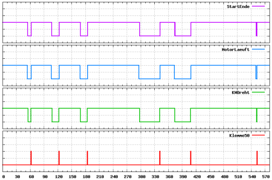

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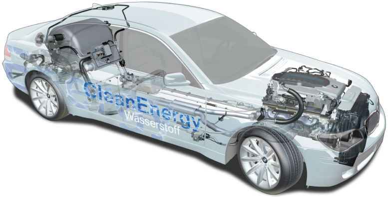

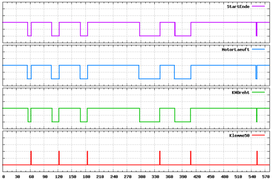

$$\Box(\mathit{PEDAL} \rightarrow \Diamond\, \mathit{BRAKE})$$

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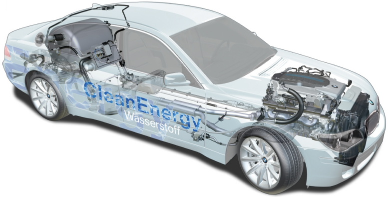
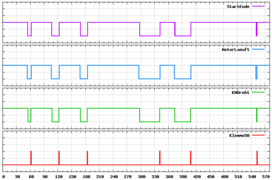




$$\square(PEDAL \rightarrow \Diamond\, BRAKE)$$

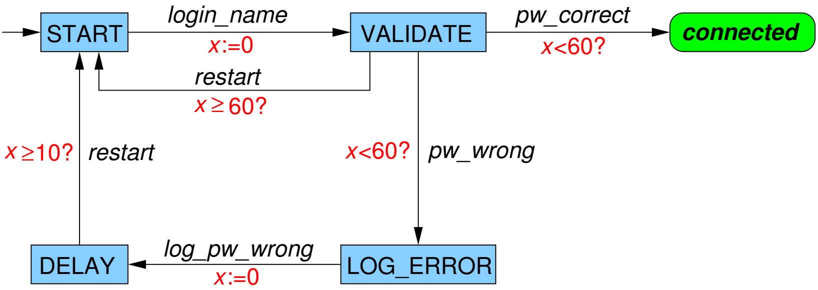
$$\square(PEDAL \rightarrow \Diamond_{[25,40]}\, BRAKE)$$

Timed Automata

Introduced by Rajeev Alur at Stanford during his PhD under David Dill:

- Rajeev Alur, David L. Dill: *Automata For Modeling Real-Time Systems*. ICALP 1990: 322-335
- Rajeev Alur, David L. Dill: *A Theory of Timed Automata*. TCS 126(2): 183-235, 1994

Timed Automata



Timed Automata

Time is modelled as the non-negative reals, $\mathbb{R}_{\geq 0}$.

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Reachability is decidable (in fact PSPACE-complete).

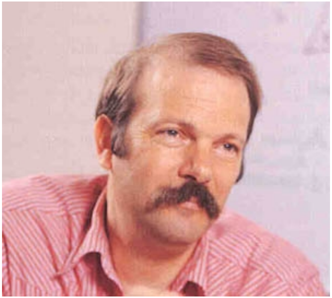
Unfortunately:

Theorem (Alur & Dill 1990)

Language inclusion is undecidable for timed automata.

Temporal Logic Model Checking

"The paradigmatic idea of the automata-theoretic approach to verification is that we can compile high-level logical specifications into an equivalent low-level finite-state formalism."



Moshe Vardi

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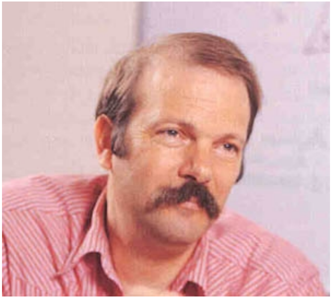
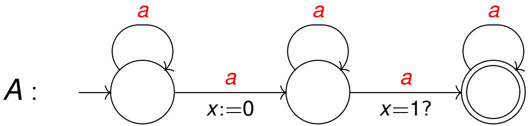


Moshe Vardi

Theorem

Automata are closed under all Boolean operations. Moreover, the language inclusion problem [$L(A) \subseteq L(B)$?] is decidable.

An Uncomplementable Timed Automaton



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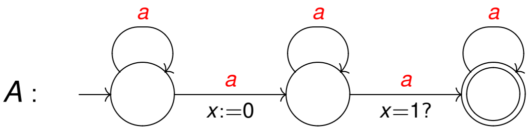
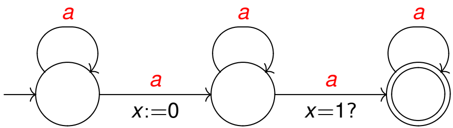


An Uncomplementable Timed Automaton

A :



L(A):

An Uncomplementable Timed Automaton

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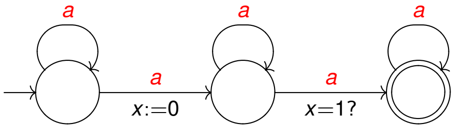
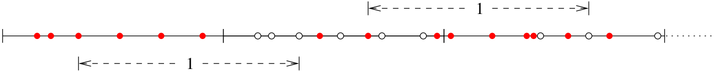


L(A):

$\overline{L(A)}$:



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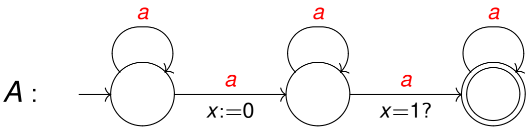

$L(A)$:

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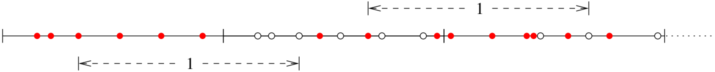

A cannot be complemented:
There is no timed automaton *B* with $L(B) = \overline{L(A)}$.

Metric Temporal Logic

$$\Box(a \rightarrow \Diamond_{[0,1]} b)$$

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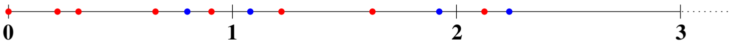

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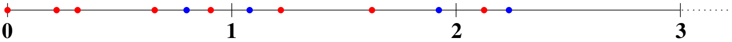

Does the timed word satisfy the specification?

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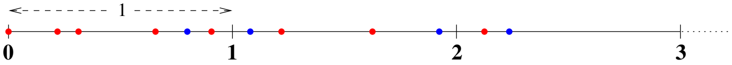

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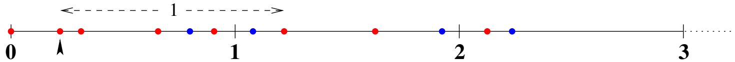

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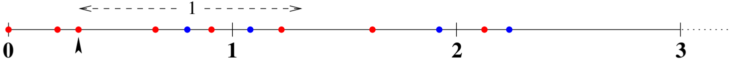

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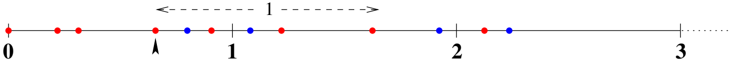

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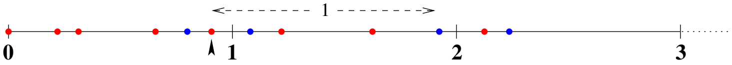

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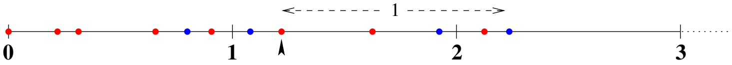

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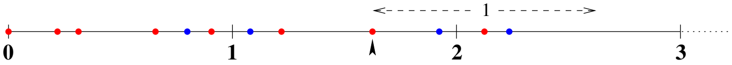

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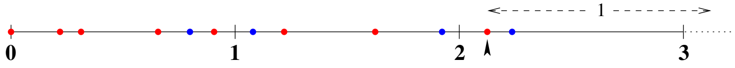

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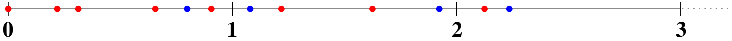

Does the timed word satisfy the specification? Yes.

Real-Time Model Checking

Given a timed automaton A and a Metric Temporal Logic specification φ, does every timed word of A satisfy φ?

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- For about 15 years ($\sim$ 1990–2005), the real-time model-checking problem was widely claimed in the literature to be undecidable.
- In 2005, James Worrell and I showed decidability through the development of the theory of timed alternating automata.


FBI
FBI


NICOLAS CAGE
JULIANNE MOORE
JESSICA BIEL
NEXT
"A CLEVER, ACTION-PACKED RIDE!"

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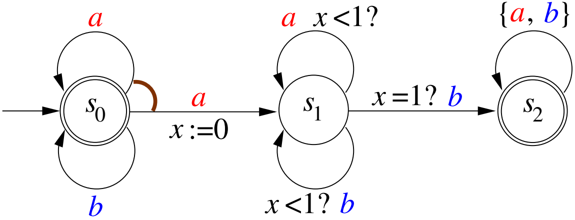

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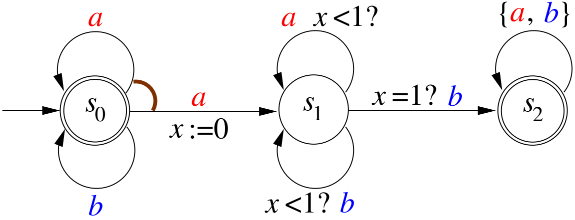

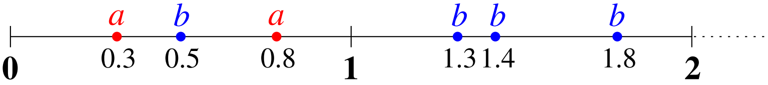

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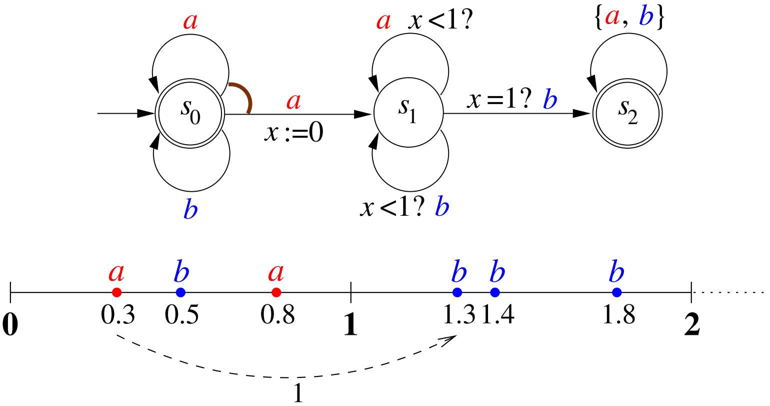

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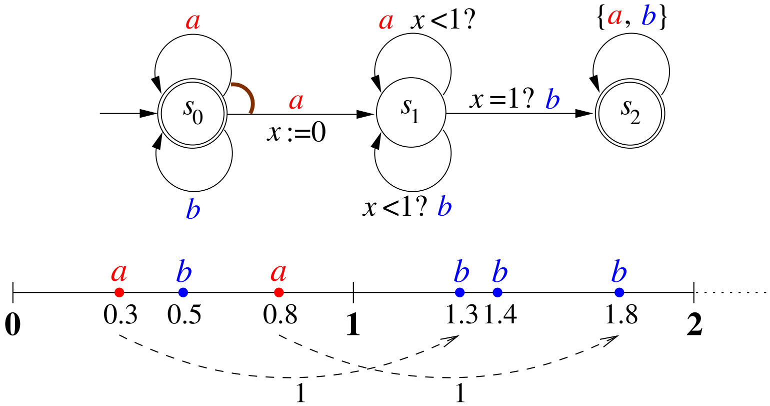

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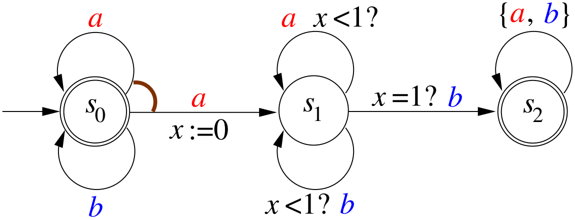

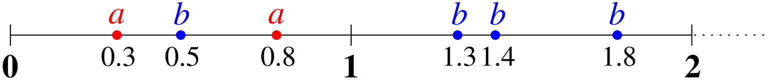

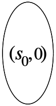

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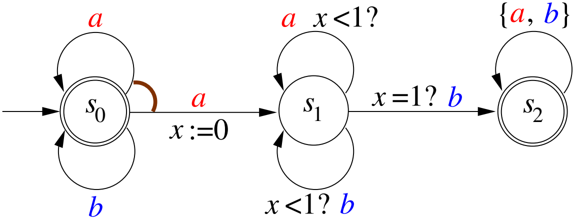

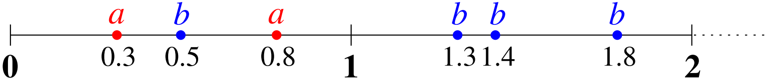

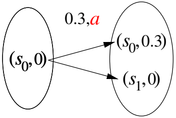

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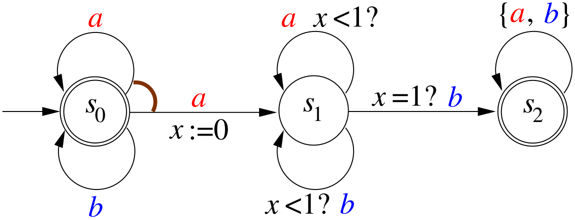

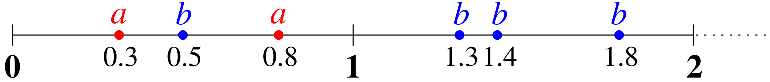

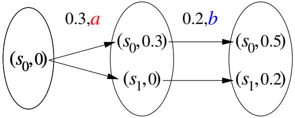

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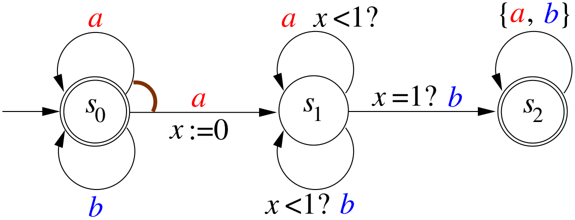
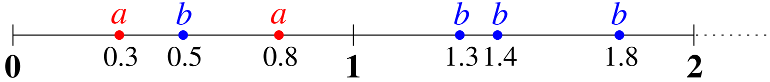
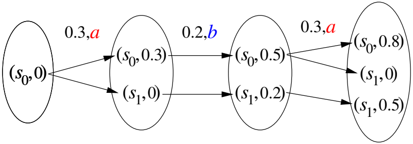
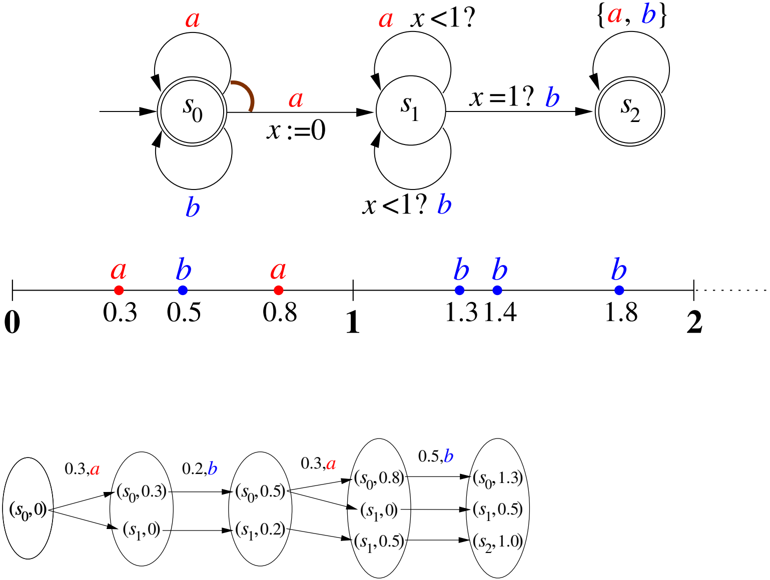






$$\Box(a \rightarrow \Diamond_{=1} b)$$



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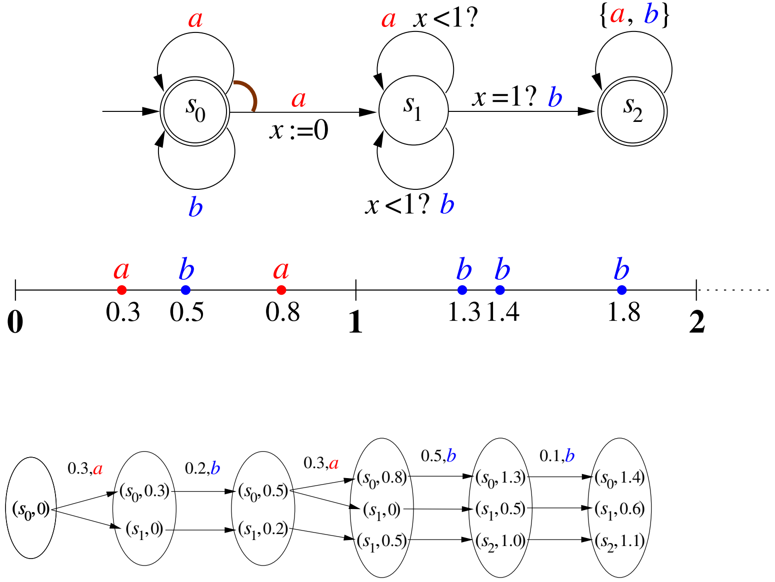

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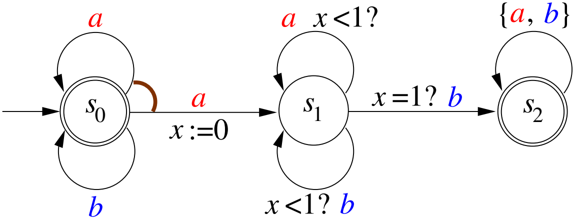

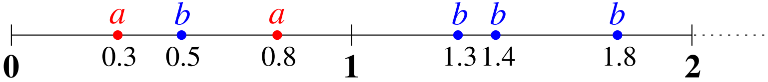

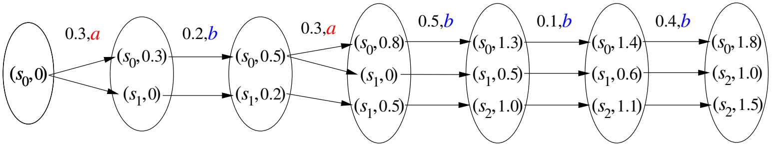

Real-Time Model Checking: A High-Level Algorithm

Real-time model checking problem

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Real-time model checking problem

$\Downarrow$

Alternating timed automaton emptiness problem

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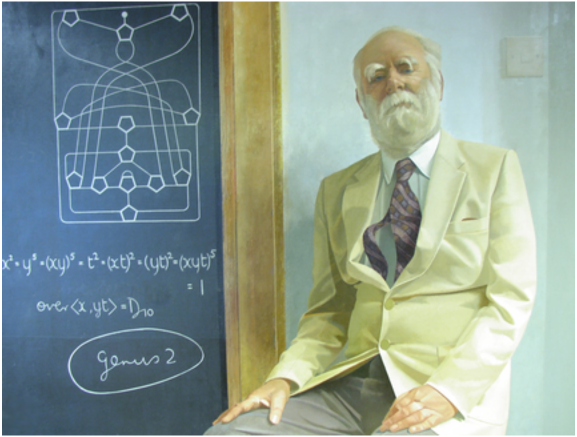
Halting problem for Turing machine with insertion errors


x²·y⁵·(xy)⁵·t²·(xt)²·(yt)²·(xyt)⁵
= 1
over ⟨x, yt⟩ = D10
genus 2

Higman's Lemma

Theorem

The subword order over a finite alphabet is a well-quasi order.

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The Halting Problem for Faulty Turing Machines

$(q_0 , \text{<>})$

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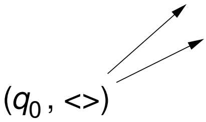

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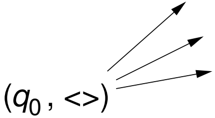

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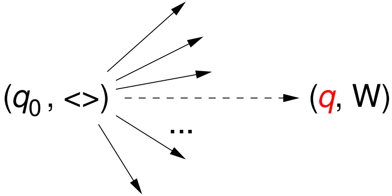

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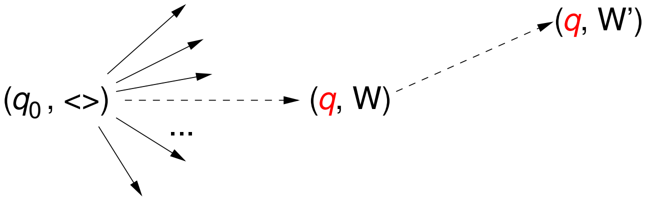

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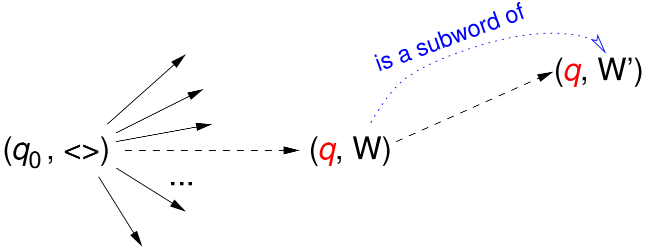

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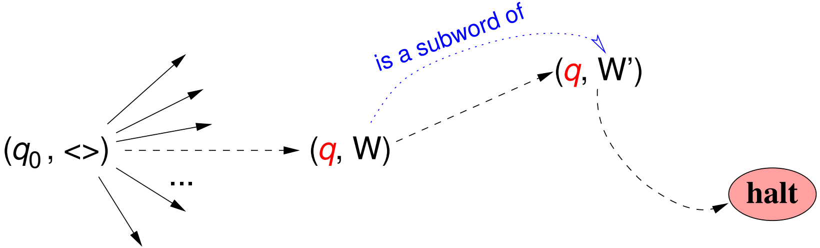

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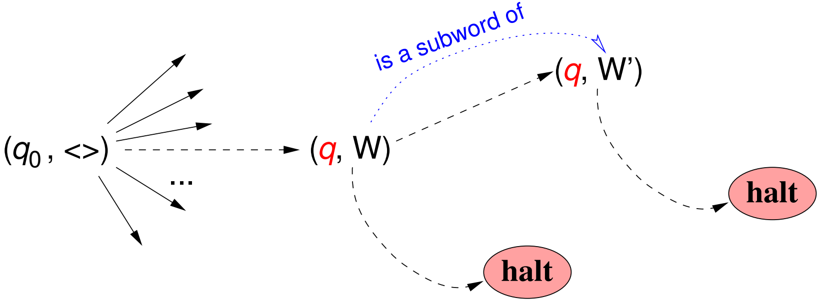

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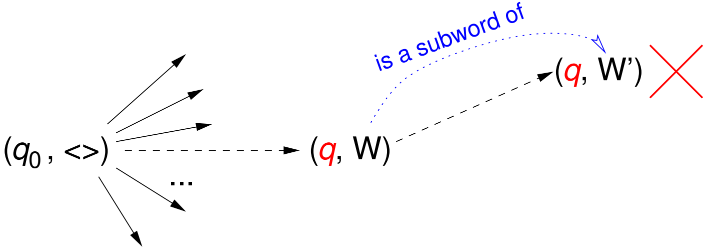

Real-Time Model Checking

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The real-time model-checking problem for Metric Temporal Logic is decidable (under the pointwise semantics).

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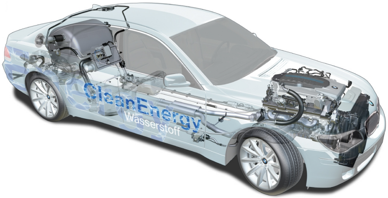
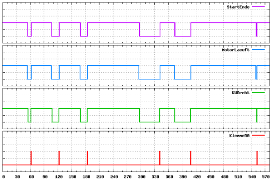
The complexity is provably non-primitive recursive. In particular, it grows faster than Ackermann's function in the worst case.

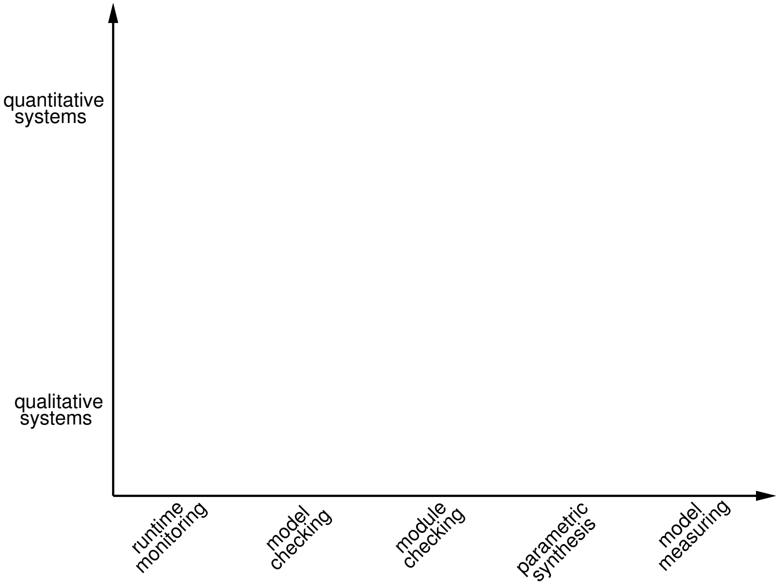
From Timed Alternating Automata to Efficient Runtime Monitoring Algorithms





$$\Box(\mathit{PEDAL} \rightarrow \Diamond_{[25,40]}\ \mathit{BRAKE})$$

Quantitative Verification: From Model Checking to Model Measuring



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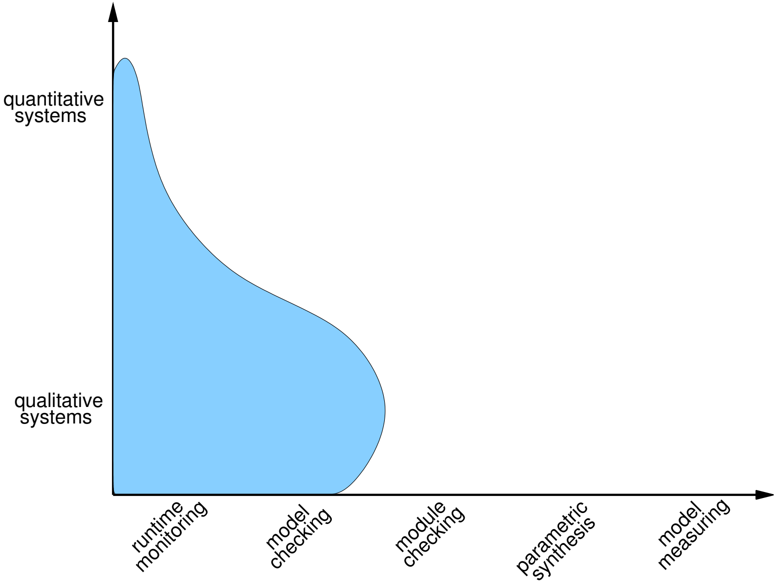

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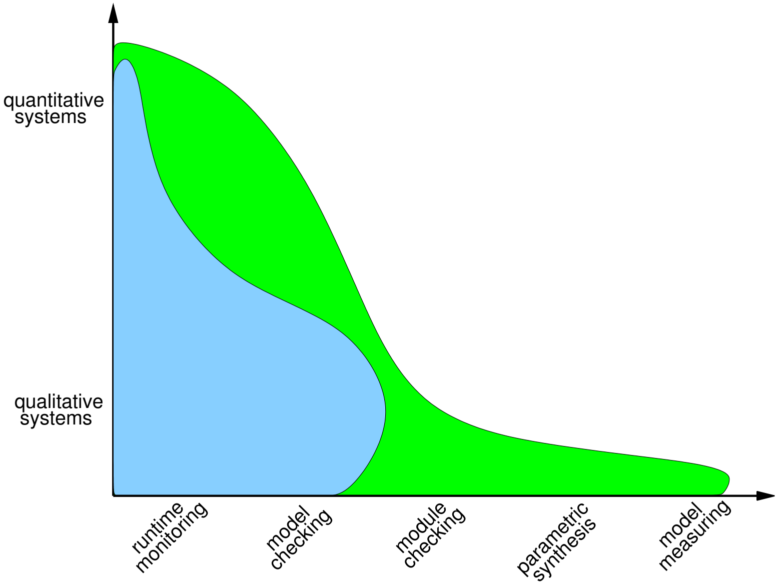

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